

SYSTEMATIC ERRORS IN NOISE PARAMETER DETERMINATION DUE TO IMPERFECT SOURCE IMPEDANCE MEASUREMENT*

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Abstract

We present a rigorous analysis of systematic errors in the four-noise parameter determination of a two-port network using the cold-source technique. The method is based on an original model that accounts for imperfectly-measured source reflection coefficients due to residual errors within a vector network analyzer (VNA). We show that even small VNA errors may seriously deteriorate the accuracy of the noise parameters when characterizing low-noise pseudomorphic high-electron mobility transistors (PHEMTs).

Introduction

Determination of the four noise parameters of a two-port network relies typically on fitting a noise model to the two-port's output noise powers measured for several source terminations whose impedances (reflection coefficients) and noise temperatures are known [1]-[3]. This determination is, however, inexact due to errors in the VNA measurement or noise temperature calibration.

Analyzing how the errors propagate in this model is not easy due to the nonlinear functional dependence of the parameters on the noise measurements. Usually, the errors are assumed random and analyzed with Monte Carlo simulations [2],[4] or a small-change sensitivity approach [5]. However, effects of correlated (systematic) measurement errors have not been studied.

Systematic source impedance measurement errors are one of the important factors that need to be considered. These errors result from residual VNA errors due to imperfections in the calibration standards, random measurement errors during the calibration and/or any change in the measurement conditions after the calibration.

This paper presents a rigorous analysis of the systematic errors in the four-noise parameter determination based on an original measurement model accounting for systematic errors in the source reflection coefficient measurements. The analysis regards the complex noise characterization [3] based on cold-source noise power measurements [1]. Results presented refer to the noise parameters of a PHEMT.

Approach

Our analysis focuses on the dependence of the effective input noise temperature T_e on the source reflection coefficient Γ_g [3], characterized by the four noise parameters: T_{eo} , T_N , $\text{Re}\Gamma_{on}$ and $\text{Im}\Gamma_{on}$, where T_{eo} is the minimum value of $T_e(\Gamma_g)$ occurring at the optimum source reflection coefficient Γ_{on} and T_N represents the slope of changes $T_e(\Gamma_g)$ away from the minimum. The parameters are determined by use of a standard noise measurement model, but this propagates errors due to the source reflection coefficients not being known exactly.

Due to residual VNA errors, the measured source reflection coefficient Γ'_g usually differs from its true value Γ_g . This can be expressed with the bilinear formula

$$\Gamma'_g = \frac{e_1 \Gamma_g + e_2}{1 - e_3 \Gamma_g}, \quad (1)$$

where e_i ($i=1,2,3$) represent the residual VNA errors. After a valid calibration, the errors are small and satisfy the following conditions:

$$e_1 \approx 1, \quad |e_2| \ll 1, \quad \text{and} \quad |e_3| \ll 1.$$

We replace conversion (1) with two fractional ones,

$$\begin{aligned} \Gamma''_g &= \frac{\Gamma_g - e_3^*}{1 - e_3 \Gamma_g} \\ \Gamma'_g &= r \Gamma''_g + c, \end{aligned} \quad (2)$$

where Γ''_g is an intermediate reflection coefficient, with

$$r = \frac{e_1 + e_2 e_3}{1 - |e_3|^2} \quad \text{and} \quad c = \frac{e_2 + e_1 e_3^*}{1 - |e_3|^2}$$

being the center and the complex radius of the unit circle $|\Gamma_g|=1$ in the complex Γ'_g plane. The decomposition of (1) into two consecutive conversions (2) facilitates analysis of the error propagation to the noise parameters.

We utilize the relationships (2) to derive a novel model describing the dependence of the noise power level p measured at the output on the erroneous Γ'_g and the noise temperature T_g of the source. We express the model in matrix notation using the quadratic form [6], as

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$$\Gamma_g^\dagger \mathbf{H} (\mathbf{T}' + T_g \mathbf{K} - p T'_r \Gamma_i' \Gamma_i'^\dagger) \mathbf{H} \Gamma_g' = 0, \quad (3)$$

where \mathbf{T}' is the transformed noise correlation matrix of the two-port, T'_r represents a system constant expressed as temperature, the matrices given in (3) are defined by

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{K} = \frac{1}{|r|} \begin{bmatrix} |r|^2 - |c|^2 & -c \\ -c^* & -1 \end{bmatrix},$$

and the vectors in (3) are defined by

$$\Gamma_g' = \begin{bmatrix} 1 & \Gamma_g' \end{bmatrix} \quad \text{and} \quad \Gamma_i' = \begin{bmatrix} 1 & \Gamma_i'^* \end{bmatrix},$$

with Γ_i being the input reflection coefficient, and the superscripts $*$ and \dagger denoting the complex and Hermitian conjugates respectively. Eq. (3) enables us to analyze the error propagation in the noise model.

Error analysis

We then studied how the fractional transformations (2) affect the noise parameters. The first of them denotes a lossless transformation of Γ_g , resulting in a change to the normalization impedance. As such, it alters only Γ_{on} but not T_{eo} , nor T_N . The alteration is negligible for small e_3 .

More serious effects result from the second conversion in (2). For the complex noise characterization [3], using the cold-source noise power measurements [1], we have derived from (3) the resultant correlation matrix

$$\tilde{\mathbf{T}}' = \frac{1}{|r|} \mathbf{T}' + T_a \left(\frac{1}{|r|} \mathbf{K} - \mathbf{H} \right), \quad (4)$$

where T_a is the ambient temperature.

We used (4) to assess how the noise parameters of a low-noise microwave amplifier and a 160 μm gate length PHEMT depend on c and r . We found that the second term of (4) represents the main error factor. Results of our calculations, made at 2 GHz on the nominal parameters of the PHEMT, with $T_{eo}=18.3$ K, $T_N=36.0$ K and $\Gamma_{on}=0.874 \angle 11.8^\circ$, are shown in Fig. 1 for T_{eo} and in Fig. 2 for Γ_{on} . They evidence high sensitivity of the noise parameters to even small residual VNA errors.

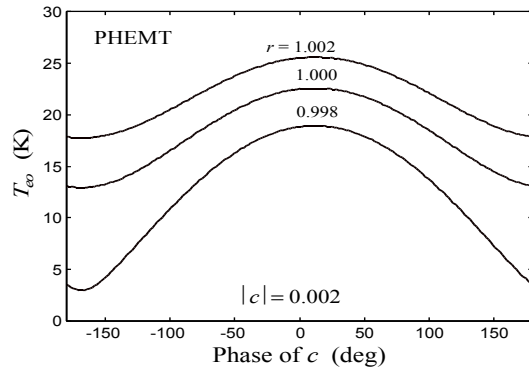


Fig. 1. Dependence of T_{eo} on the phase angle of c with r as parameter.

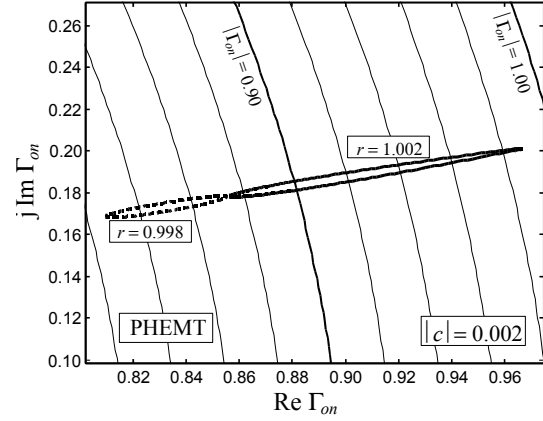


Fig. 2. Loci curves $|c|=0.002$ drawn for the PHEMT in the complex Γ_{on} plane with r as parameter.

Conclusions

We presented a rigorous analysis of systematic measurement errors in the four-noise parameter determination caused by VNA residual errors in the source impedance measurement. We applied this analysis to a novel noise measurement model describing dependence of the two-port's output noise power on the noise temperature and erroneous source reflection coefficient. We performed our analysis by decomposition of the residual VNA errors into two groups, each relevant to different error factors. The analysis revealed that even small residual VNA errors seriously alter the noise parameters of a low-noise PHEMT. This result explains difficulties encountered when characterizing those devices at low microwave frequencies using the cold-source technique.

References

- [1] V. Adamian, A. Uhler, "Simplified noise evaluation of microwave receivers," *IEEE Trans. Instrum. Meas.*, vol. IM-33, pp. 487-490, June 1984.
- [2] A.C. Davidson, *et al.*, "Accuracy improvements in microwave noise parameter measurements," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-37, pp. 1973-1978, Dec. 1989.
- [3] W. Wiatr, "Characterization of radiometer using eight-term linear model," *IEEE Trans. Instrum. Meas.*, vol. IM-44, No. 2, pp. 343-346, April 1995.
- [4] J. Randa, W. Wiatr, "Monte Carlo estimation of noise-parameter uncertainties," *IEE Proc.-Sci. Meas. Technol.*, vol. 149, pp. 333-337, Nov. 2002.
- [5] M. Schmidt-Szalowski, W. Wiatr, "A method for evaluation of uncertainties of noise parameter measurement", XIII Int. Microwave Conf. MIKON-2000, Wroclaw, Poland, pp. 539-542, May 2000.
- [6] W. Wiatr, "Multi-state radiometry: noise de-embedding and un-terminating," *IEEE Trans. Instrum. Meas.*, vol. IM-46, pp. 503-506, April 1997.